Lecture 5

Structure Functions at Low Q^2

and

the CQ Picture

- CQ's as intermediate structures between current quarks and hadrons → two-stage model

  O.K. with DIS data

- **extension of the two-stage model at low Q^2** [Petronzio et al. ('03)]

  as Q^2 decreases below $\approx \mu_0^2 \approx 1\,GeV^2$ we expect that:

  1) the inelastic coupling of CQ's with γ^* becomes less and less important;

  2) the elastic coupling of CQ's with γ^* becomes more and more important.

- CQ structure function: $F_2^j = F_2^{j(inel.)} + F_2^{j(el.)}$

$$F_2^H(x, Q^2) = \sum_j \int_x^1 dz\, f_j^H(z)\, F_2^j\left(\frac{x}{z}, Q^2\right)$$

$$F_2^H(x, Q^2) = \sum_j \int_x^1 dz\, f_j^H(z)\, F_2^{j(inel.)}\left(\frac{x}{z}, Q^2\right) + \sum_j \int_x^1 dz\, f_j^H(z)\, F_2^{j(el.)}\left(\frac{x}{z}, Q^2\right)$$

naïve expectation !

- **elastic channel at CQ level:** $F_2^{j(el.)}(x) = \left[G_j(Q^2)\right]^2 \delta(x-1)$

where $\left[G_j(Q^2)\right]^2 \equiv \left[F_1^j(Q^2)\right]^2 + \tau\left[F_2^j(Q^2)\right]^2$ $\qquad \tau = Q^2/4m_j^2$

$$F_2^H(x,Q^2) = \sum_j \int_x^1 dz\, f_j^H(z)\, F_2^{j(inel.)}\left(\frac{x}{z},Q^2\right) + \sum_j \left[G_j(Q^2)\right]^2 x \cdot f_j^H(x)$$

- DIS regime: $F_2^H(x,Q^2) \approx \sum_j \int_x^1 dz\, f_j^H(z)\, F_2^{j(inel.)}\left(\frac{x}{z},Q^2\right)$

- $0.1 \div 0.2 < Q^2\ (\mathrm{GeV}^2) < 1 \div 2$: $\quad F_2^H(x,Q^2) \approx \sum_j \left[G_j(Q^2)\right]^2 x \cdot f_j^H(x)$

  $\approx \Lambda^2_{QCD}$ $\qquad\qquad \approx \Lambda^2_{\chi}$

it cannot hold at each x value !

- Cornwall-Norton moments: $M_n^H\left(Q^2\right) = \int_0^1 dx\, x^{n-2}\, F_2^H\left(x,Q^2\right)$

- dual moments: $M_n^{dual}\left(Q^2\right) = \int_0^1 dx\, x^{n-2} \sum_j \left[G_j\left(Q^2\right)\right]^2 x \cdot f_j^H(x)$

CQ-hadron duality: $M_n^H\left(Q^2\right) \approx M_n^{dual}\left(Q^2\right)$ for low values of n, but n > 2

- squared CQ form factor: $\left[F\left(Q^2\right)\right]^2 = \frac{\sum_j \left[G_j\left(Q^2\right)\right]^2}{\sum_j e_j^2} = \frac{\sum_j \left[F_1^j\left(Q^2\right)\right]^2 + \tau\left[F_2^j\left(Q^2\right)\right]^2}{\sum_j e_j^2}$

SU(2) symmetric form factors: $M_n^{dual}\left(Q^2\right) = \left[F\left(Q^2\right)\right]^2 \cdot \overline{M}_n^H$

where $\overline{M}_n^H = \int_0^1 dx\, x^{n-1} \sum_j e_j^2\, f_j^H(x)$

- **define:** $R_n^H\left(Q^2\right) \equiv M_n^H\left(Q^2\right) / \overline{M}_n^H$

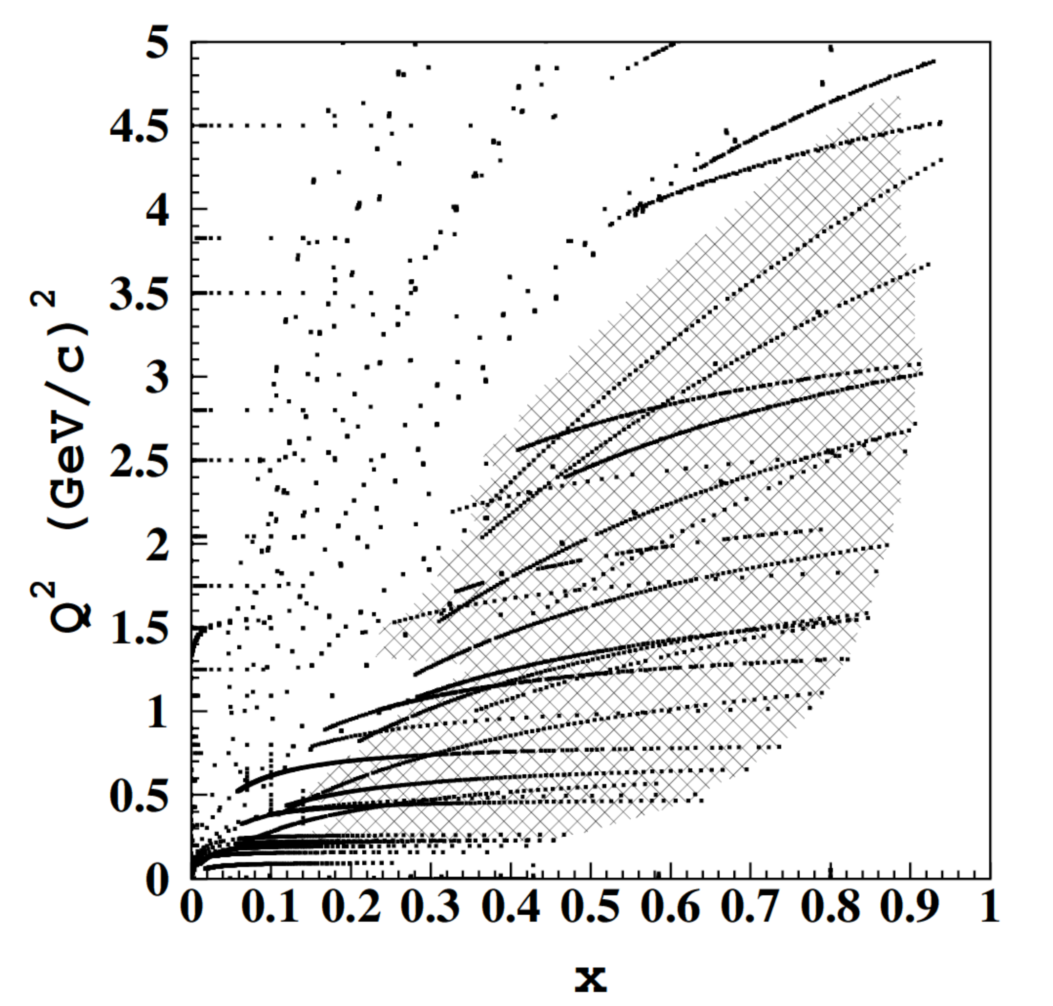
CQ picture ⇨ CQ-hadron duality ⇨ $R_n^H\left(Q^2\right) \sim \left[F\left(Q^2\right)\right]^2$

scaling property: the ratio becomes independent on n

scaling function: the squared CQ form factor (independent also on H)

Note: once the CQ form factor is extracted from known data on the hadron H, using a reasonable model for $f_j^{H'}(z)$ one can predict the low-order moments of another hadon H'

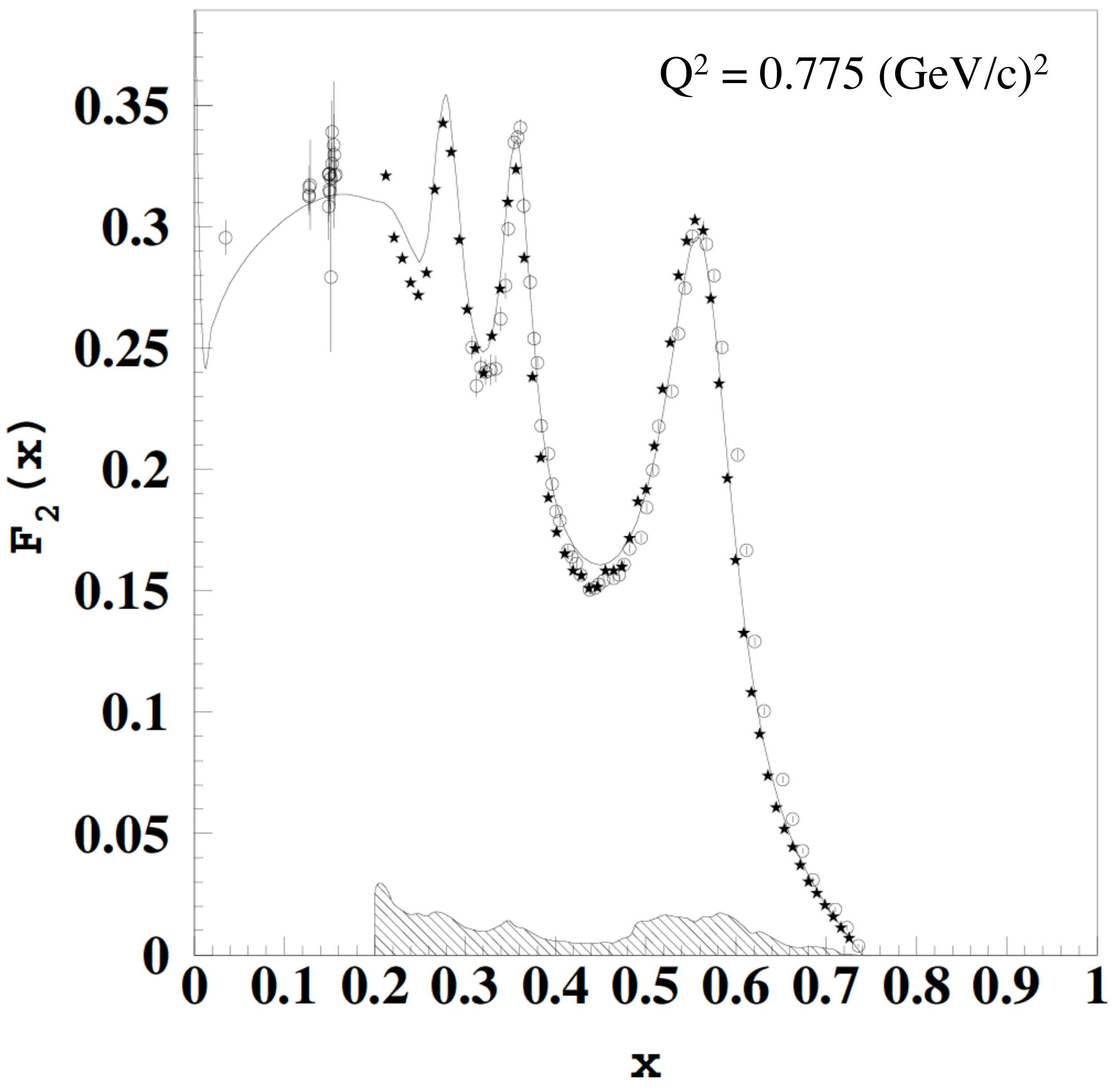
- **CLAS data**: map of F_2 of the proton for $W < 2.5$ GeV and $Q^2 < 4.5$ GeV^2 [M. Osipenko et al. ('03)]



kinematical coverage

shaded area: CLAS kinematics

points: previous world data for $Q^2 < 5$ (GeV/c)^2



★ CLAS

○ previous exp.'s

— param. from Ricco et al. ('99)

hatched area: systematic errors

- Nachtmann moments of the structure function

$$M_n^p(Q^2) = \int_0^1 dx \, \frac{\xi^{n+1}}{x^3} \frac{3+3(n+1)r+n(n+2)r^2}{(n+2)(n+3)} F_2^p(x,Q^2)$$

- Nachtmann variable: $\begin{cases} \xi = 2x/(1+r) \\ r = \sqrt{1+4M^2x^2/Q^2} \end{cases}$

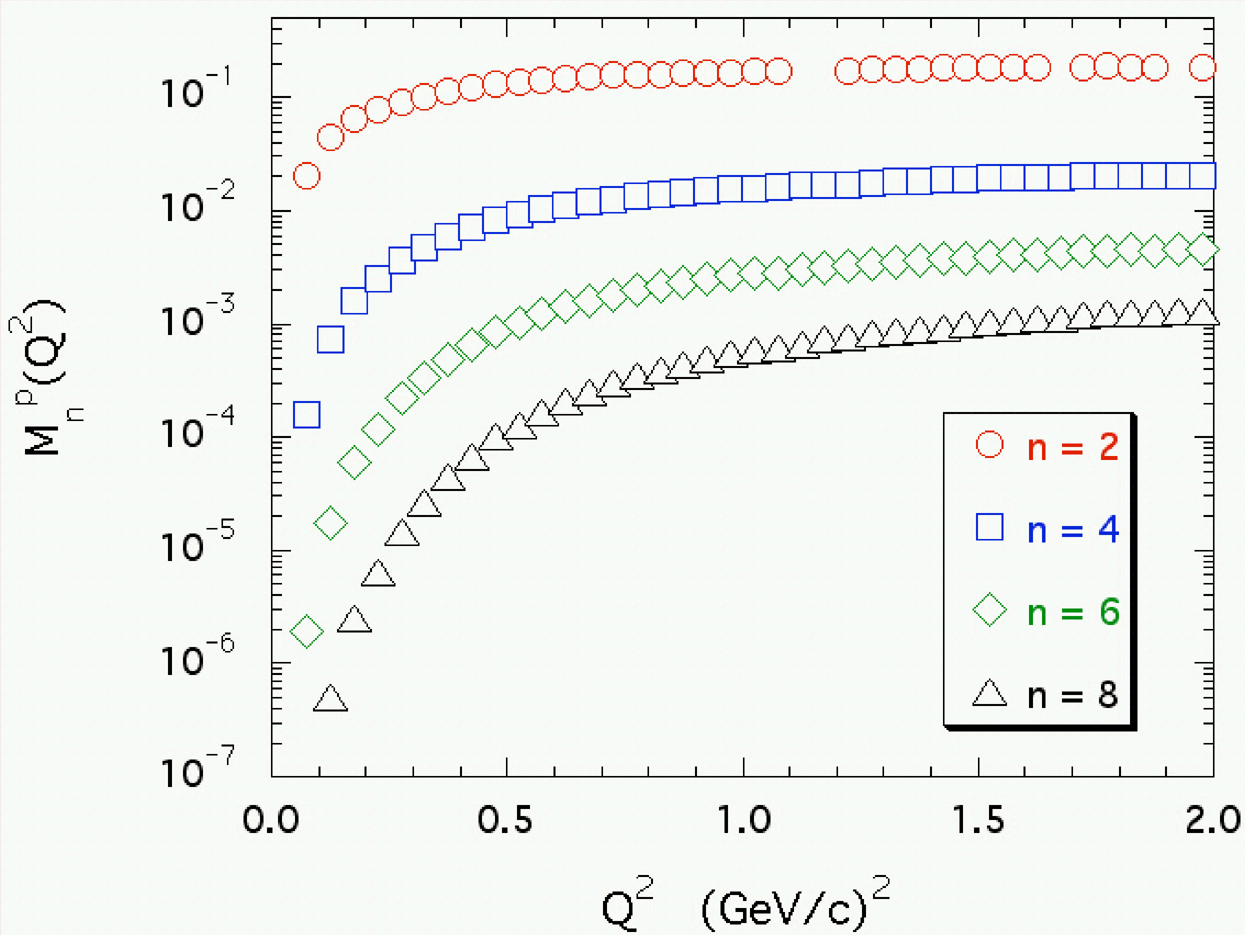
$$M_n^p(Q^2) \xrightarrow[Q^2 >> M^2]{} \int_0^1 dx \, x^{n-2} F_2^p(x,Q^2)$$

- no target-mass effects on $M_n^p(Q^2)$

$$M_n^p(Q^2) = \text{leading twist} + \text{dynamical higher twists}$$

parton correlations

CLAS data + world data → construction of experimental (> 90%) Nacthmann moments



[M. Osipenko et al. ('03)]

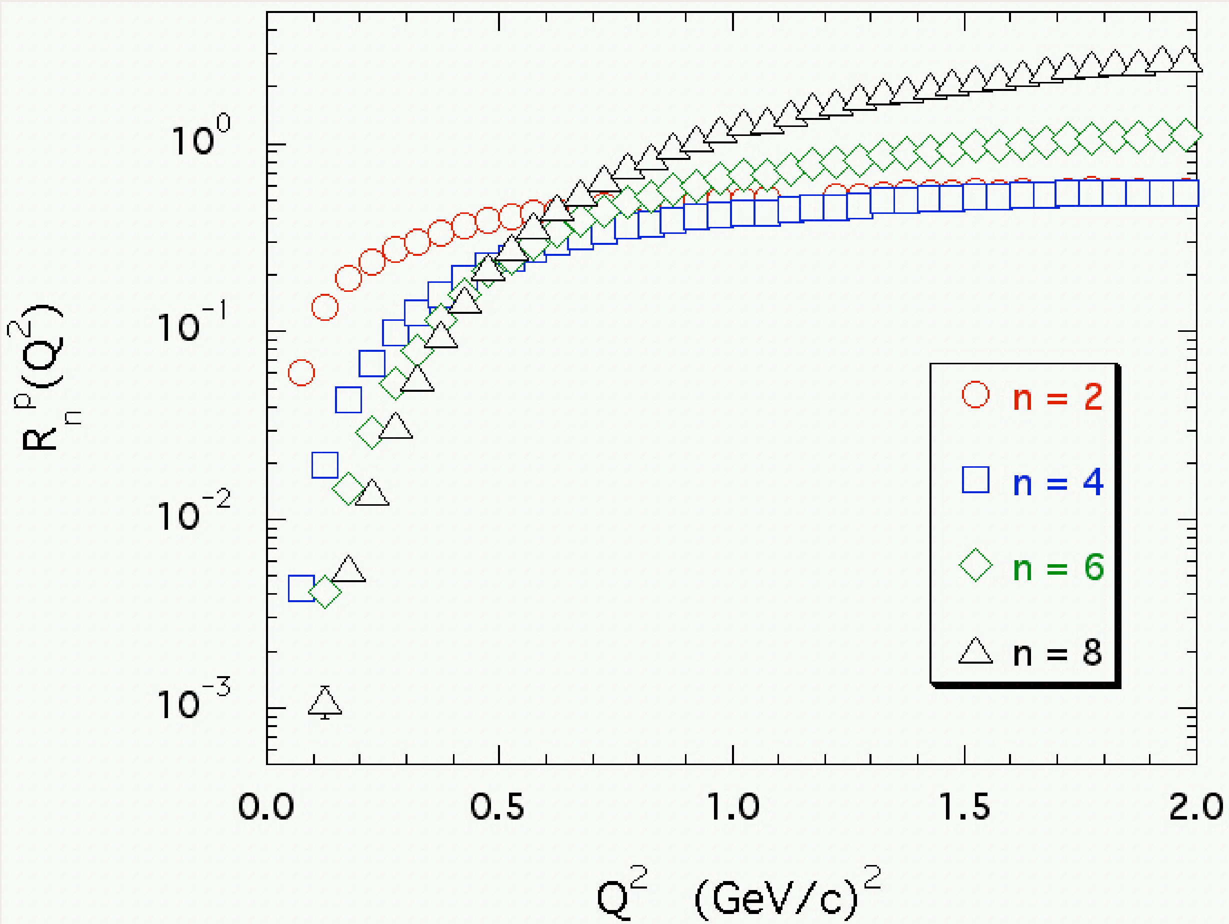
- sharp rise at low Q^2, smoother behavior for $Q^2 > 1$ GeV^2

- strong dependence on n: ~ one order of magnitude moving from n to n+2

- assume that CQ's share exactly (1/3) of the LF proton momentum:

$$\sum_j e_j^2 f_j^p(x) \propto \delta\left(x - \frac{1}{3}\right) \quad \Rightarrow \quad \overline{M}_n^p = \int_0^1 dx\, x^{n-1} \sum_j e_j^2 f_j^p(x) \propto \left(\frac{1}{3}\right)^{n-1}$$

factor (1/9) between orders n and (n+2)



- spread of values reduced
- tendency toward a scaling property

- consider the relative motion of CQ's inside the proton

$$\bar{f}^{p}(x) = \sum_j e_j^2 \, f_j^p(x) = \frac{1}{9}\left[4 f_U^p(x) + f_D^p(x)\right]$$

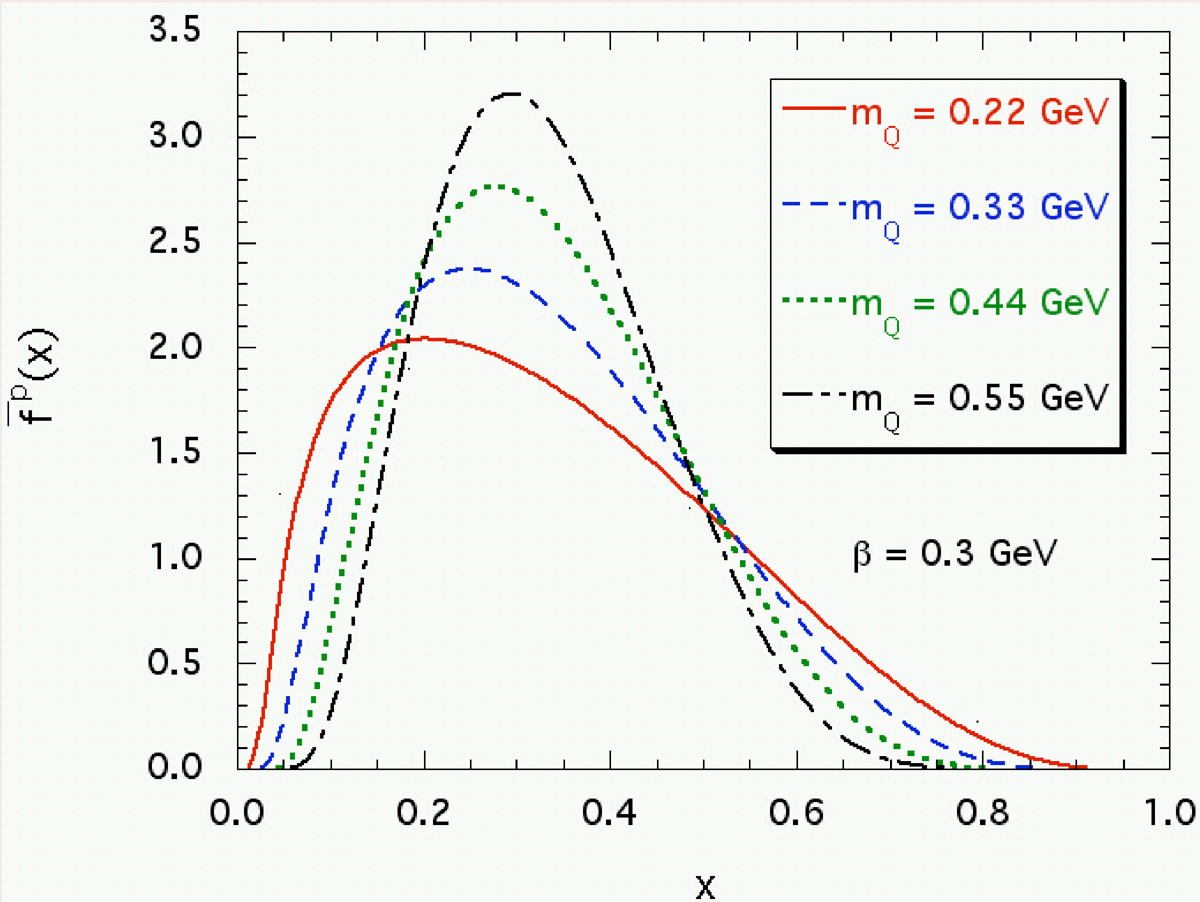
where $$f_Q^p(x) = \frac{3}{2} \sum_{\lambda_p} \int \prod \left[d\xi_i \, d\vec{k}_{i\perp}\right] \sum_{\{\nu_i \tau_i\}} \delta(x - \xi_1)\, \delta_{\tau_Q \tau_1} \left| \left\langle \{\xi_i \vec{k}_{i\perp}; \nu_i \tau_i\} \middle| \Psi_p^{\lambda_p} \right\rangle \right|^2$$

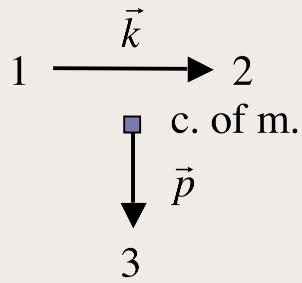
LF proton wave function

- normalizations: $\int_0^1 dx \, f_U^p(x) = 2 \quad , \quad \int_0^1 dx \, f_D^p(x) = 1$

- momentum sum rule: $\int_0^1 dx \, x \cdot \left[f_U^p(x) + f_D^p(x)\right] = 1$

- SU(6) symmetric wave function: $\bar{f}^{\,p}(x) = \int d\vec{k}_\perp d\vec{p}_\perp \int [d\xi_i]\,\delta(x-\xi_1)\,\frac{E_1E_2E_3}{M_0\xi_1\xi_2\xi_3}\left|w_S(\vec{k},\vec{p})\right|^2$

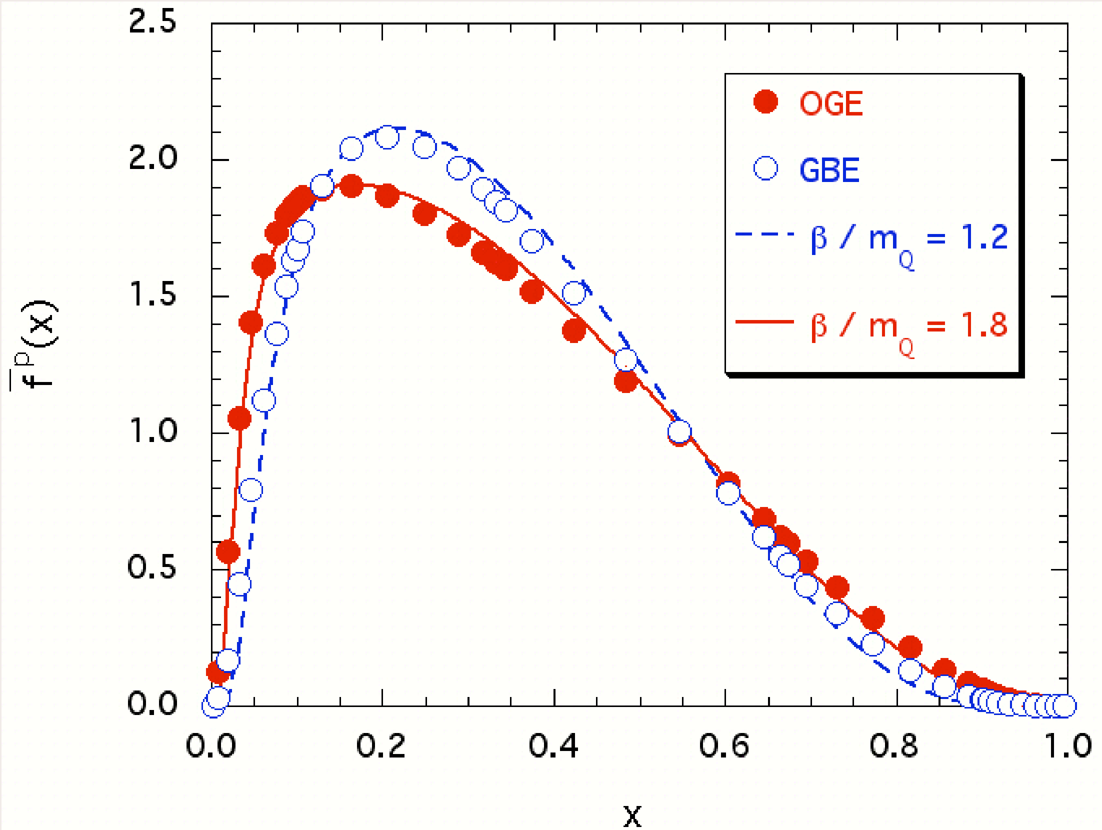




$w_S \propto e^{-\left(k^2+3p^2/4\right)/2\beta^2}$

important effect of the internal motion, depending on the ratio β / m_Q

quark potential models → SU(6) symmetry breaking



One-Gluon-Exchange model
[N. Isgur et al. (‘86)]

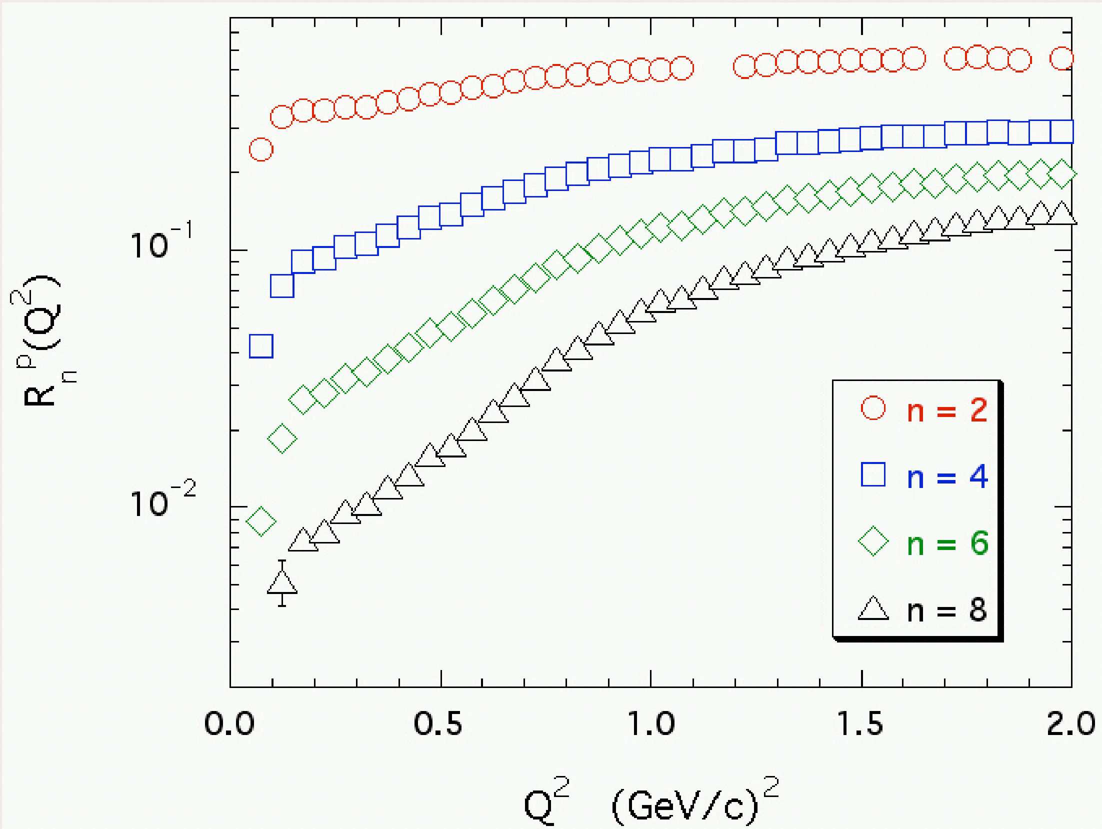
Goldstone-Boson-Exchange model
[L. Glozman et al. (‘98)]

the gaussian ansatz is a good first approximation with appropriate values of the ratio β/m_Q

□= 0.3 GeV

m_Q = 0.25 GeV

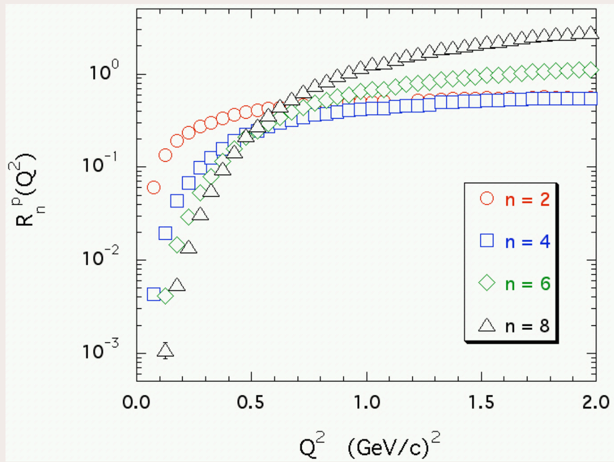
gaussian ansätz



an improvement,

but still unsatisfactory

□(x-1/3)



- the main drawback is that the equation $\overline{M}_n^H = \int_0^1 dx\, x^{n-1} \sum_j e_j^2 f_j^H(x)$ has a meaning only in the Bjorken limit

- we have to account for **power corrections**:

1) inelastic pion threshold (final-state phase-space constraints): $x_{\max} = x_\pi \equiv \dfrac{Q^2}{Q^2 + (M + m_\pi)^2 - M^2}$

2) kinematical power corrections due to the target mass M ~ 1 GeV

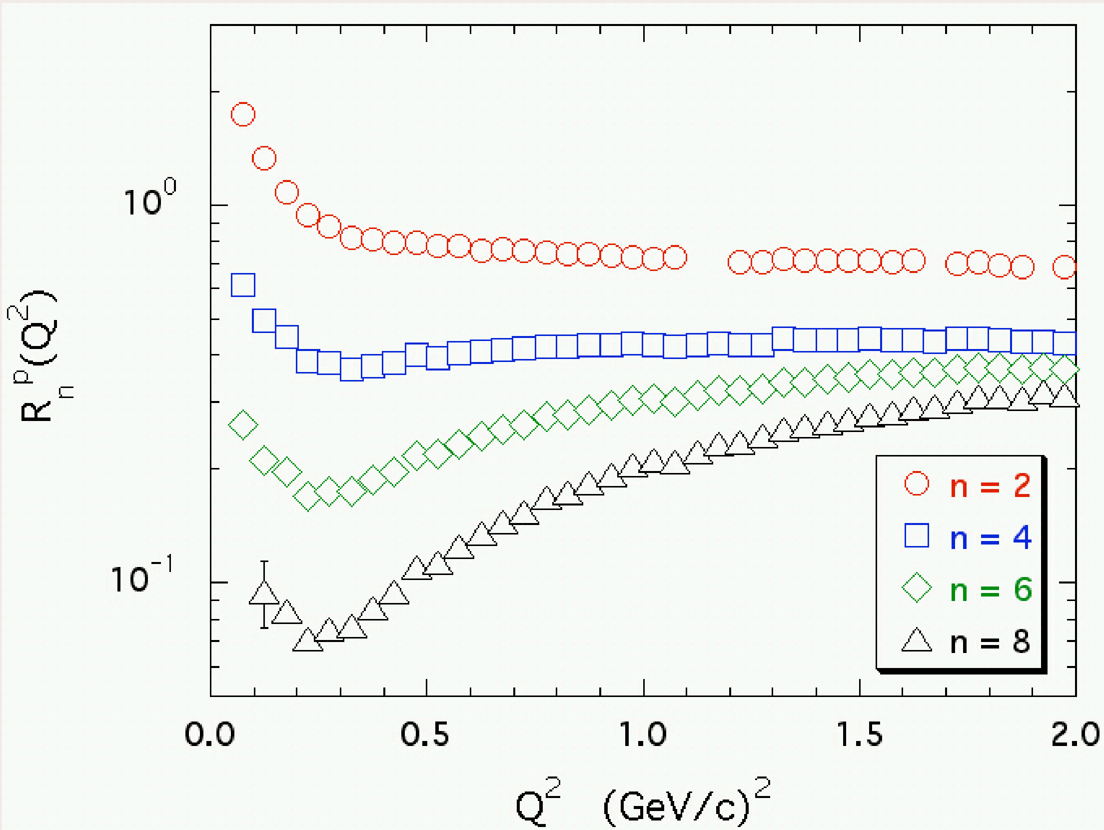
3) dynamical power corrections due to final-state interactions (responsible for resonances)

- with threshold factor: $\bar{f}^{p}(x) \to \bar{f}^{p}(x) F_{thr}(W)$

$\Lambda = 0.3$ GeV

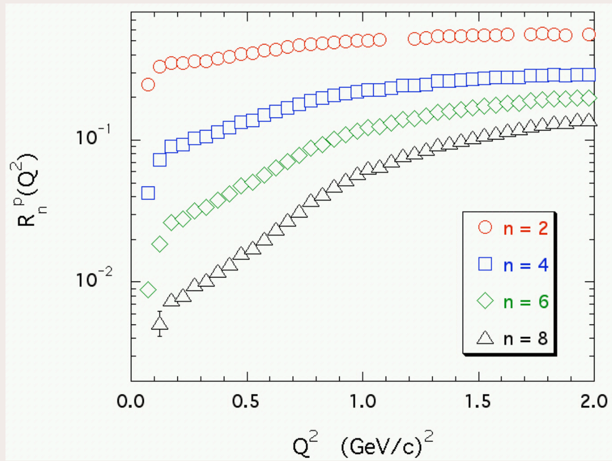
$m_Q = 0.25$ GeV

$$F_{thr}(W) = \sqrt{1 - \frac{(M + m_{\pi})^2}{W^2}}$$



a small improvement

$F_{thr}(W) = 1$



- target-mass corrections:

in analogy with the DIS case we replace $\bar{f}^{\,p}(x)$ with $\bar{f}^{\,p}_{TM}(\xi,Q^2)$, given by

$$\bar{f}^{\,p}_{TM}(\xi,Q^2)=\frac{x^2}{r^3}\frac{\bar{f}^{\,p}(\xi)}{\xi^2}+\frac{6M^2}{Q^2}\frac{x^3}{r^4}\int_{\xi}^{\xi_{\max}}d\xi'\,\frac{\bar{f}^{\,p}(\xi')}{\xi'^2}+\frac{12M^4}{Q^4}\frac{x^4}{r^5}\int_{\xi}^{\xi_{\max}}d\xi'\,\frac{\bar{f}^{\,p}(\xi')}{\xi'^2}(\xi'-\xi)$$

ξ = Nachtmann variable, $r=\sqrt{1+4M^2x^2/Q^2}$, $x=\xi\big/\left(1-M^2\xi^2/Q^2\right)$, $\xi_{\max}=\min(1,Q/M)$

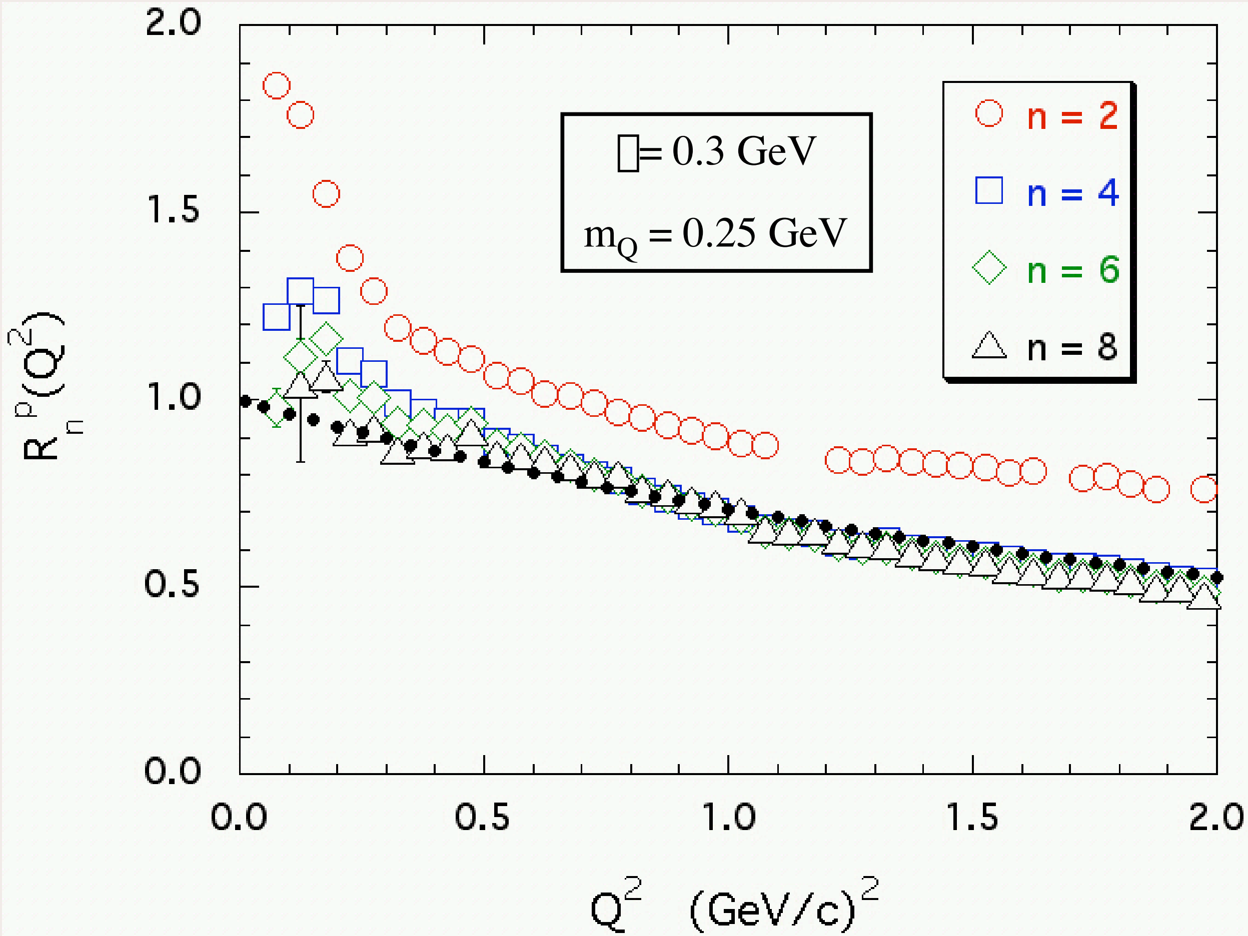
$$\bar{f}^{\,p}_{TM}(\xi,Q^2)\xrightarrow[Q^2>>M^2]{}\bar{f}^{\,p}(x)$$

- re-definition of **dual moments**: $M_n^{dual}(Q^2)=\left[F(Q^2)\right]^2\cdot\overline{M}_n^{\,p}(Q^2)$

$$\overline{M}_n^{\,p}(Q^2)=\int_0^{\xi_{\max}}d\xi\,\frac{\xi^{n+1}}{x^3}\,\frac{3+3(n+1)r+n(n+2)r^2}{(n+2)(n+3)}\,\frac{r(1+r)}{2}\,\bar{f}^{\,p}_{TM}(\xi,Q^2)\,F_{thr}(W)$$

note that when $F_{thr}(W)$ = 1 one has $\overline{M}_n^{\,p}(Q^2)\xrightarrow{}\int_0^1 dx\,x^{n-1}\,\bar{f}^{\,p}(x)=\overline{M}_n^{\,p}$

- with threshold factor and kinematical (target-mass) corrections



scaling between
~ 0.2 and ~ 2 GeV²

for n > 2

$$[F(Q^2)]^2 = \left(\frac{1}{1 + r_Q^2\, Q^2/6}\right)^2$$

$r_Q = 0.21$ fm

what is included in the model and what is not ?

- consider the OPE at the fundamental level (current quarks and gluons of QCD):

  - higher twists (HT) are matrix elements of local operators acting on elementary (point-like) fields

  - series of matrix elements of operators O_n producing terms of the form $\left(\frac{\Lambda_n^2}{Q^2}\right)^{(\tau_n-2)/2}$

  twist of the operator O_n

  Λ_n = scale proportional to the inverse of R_n

  R_n= average distance of partonic correlations generated by O_n

    a) correlations amog partons in the same CQ: $R_n < r_Q$

    b) correlations among partons belonging to different CQ's: $R_n \sim 1 / \Lambda_{QCD}$ ~ conf. size > r_Q

- short-range HT (a) are accounted for by the CQ form factor — included and relevant for $Q < \Lambda_{\square}$

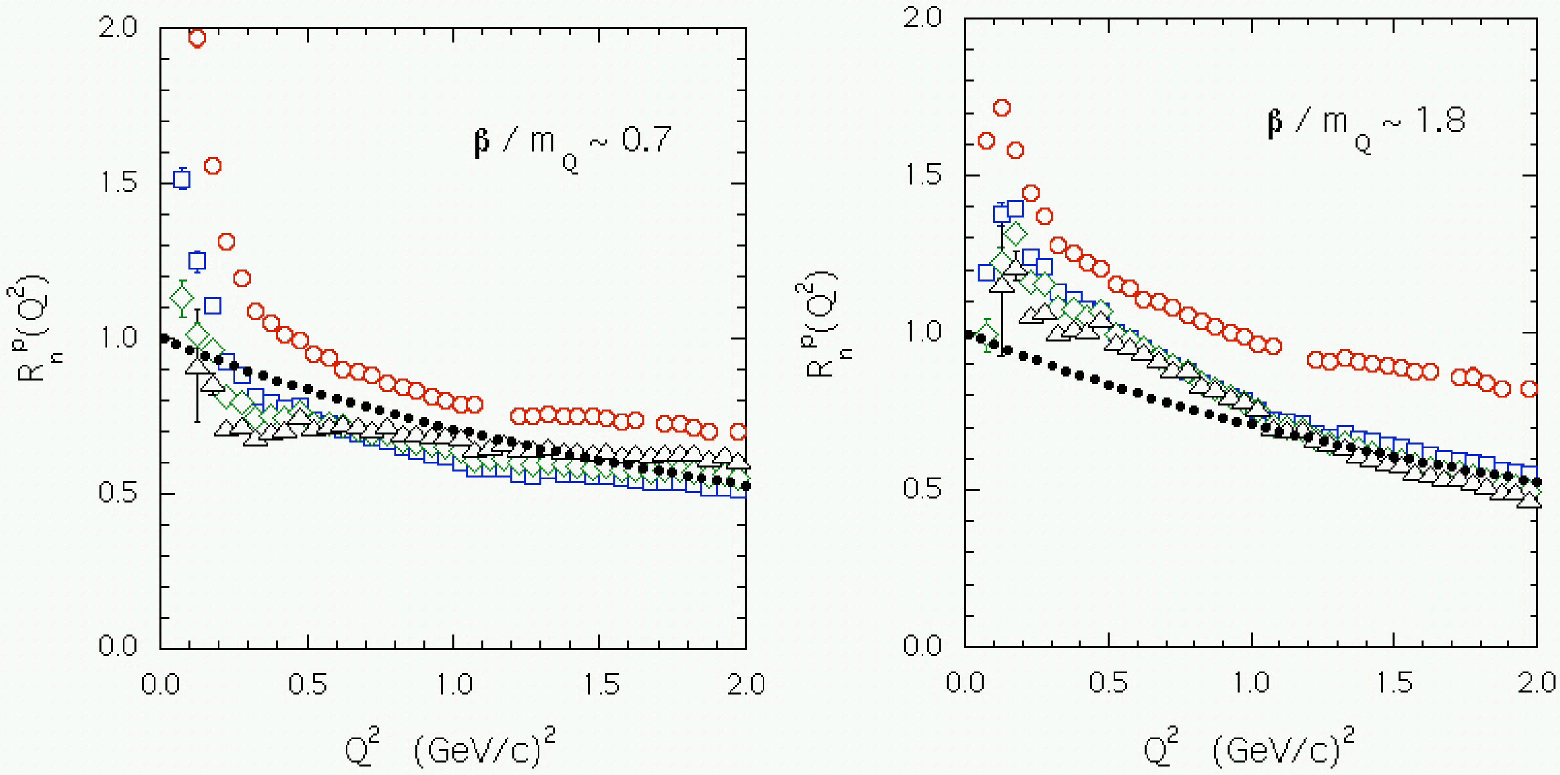
- long-range HT (b) generates the resonance bumps in the x-space — not included, but relevant only for $Q < \Lambda_{QCD}$

phenomenological inputs of the generalized two-stage model

1) the value of the ratio $\square / m_Q$;

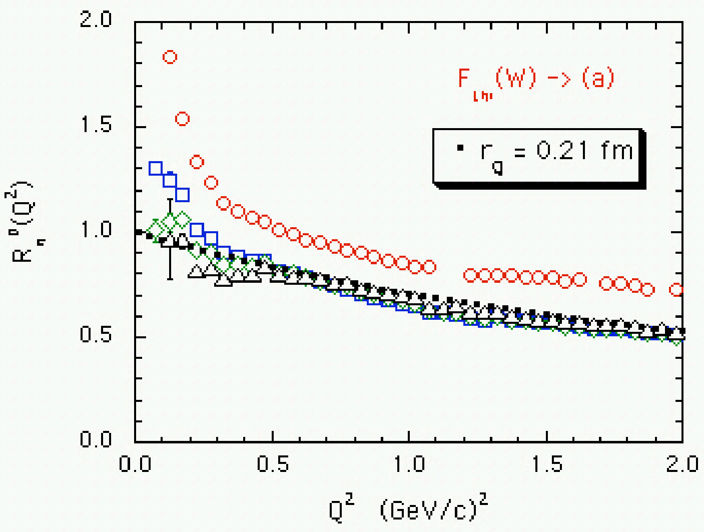
2) the shape of the threshold factor.

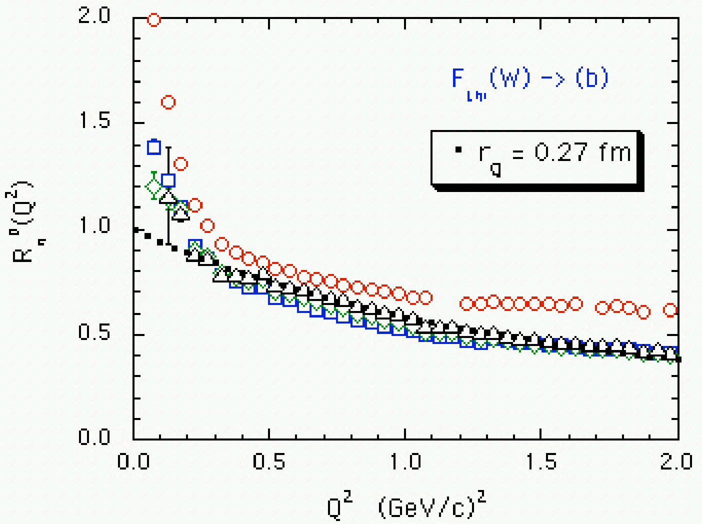
- effects of the ratio β / m_Q: .... $r_Q = 0.21$ fm ⇒ β / $m_Q = 1.2$

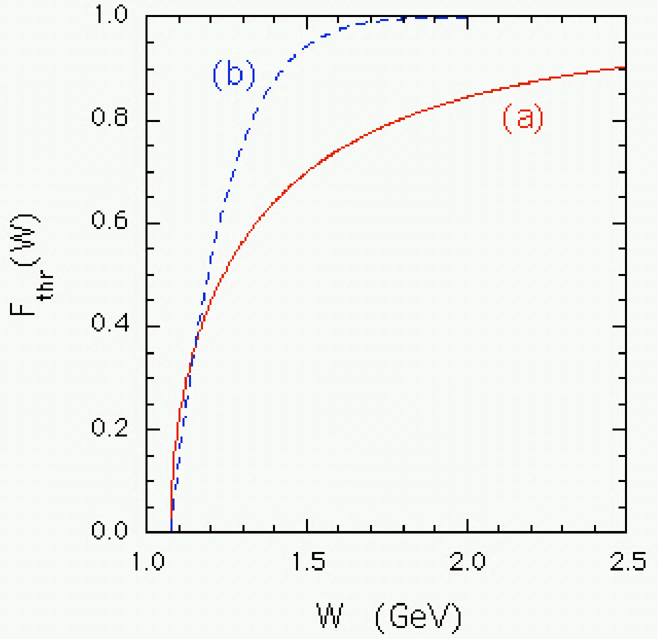


the scaling property is not affected by β / m_Q, but the scaling function is

- effects of the shape of the threshold factor:







the scaling property is not affected by the shape of $F_{thr}(W)$, but the scaling function is

CQ size ~ 0.2 ÷ 0.3 fm

- **consistency check**: reproduction of nucleon elastic data using the same CQ form factor and the same wave function

- covariant LF approach @ $q^+ = 0$:

  - one-body e.m. at the CQ level: $J^{\mu} \approx J_1^{\mu} = \sum_j \left\{ F_1^j(Q^2)\gamma^{\mu} + F_2^j(Q^2) \frac{i\sigma^{\mu\nu}q_{\nu}}{2m_j} \right\}$

$$F_1^j(Q^2) = e_j \Big/ \left(1 + r_Q Q^2/6\right)$$

$$F_2^j(Q^2) = k_j \Big/ \left(1 + r_Q Q^2/12\right)^2$$

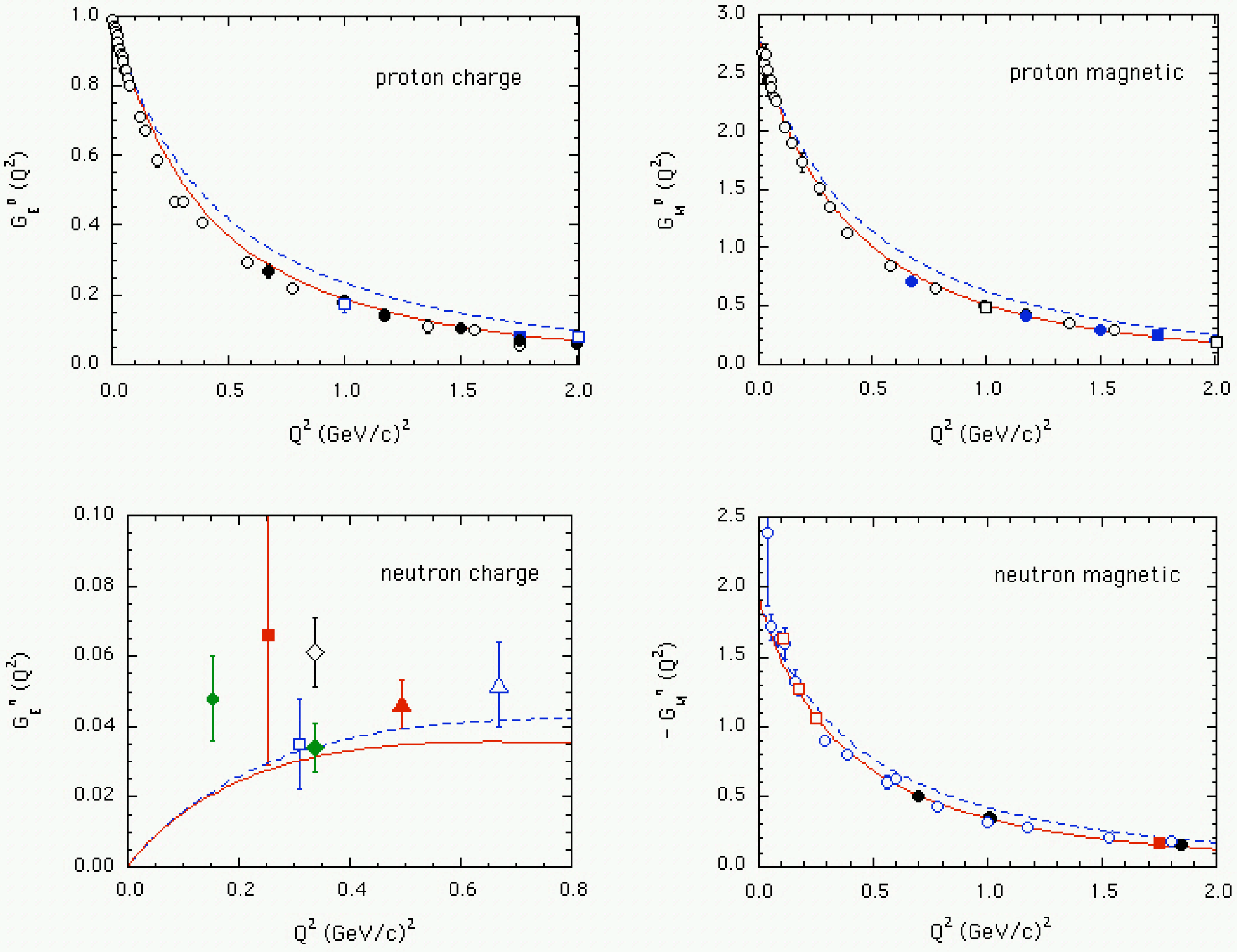
$$\Rightarrow \quad \left[F(Q^2)\right]^2 = \frac{\sum_j \left[F_1^j(Q^2)\right]^2 + \tau\left[F_2^j(Q^2)\right]^2}{\sum_j e_j^2} \approx \frac{1}{\left(1 + r_Q Q^2/6\right)^2}$$

k_j → fixed by the reproduction of μ_N

  - nucleon Sachs form factors: $G_E^N(Q^2) = \frac{1}{2} Tr\left\{ J^+ \left[1 - \frac{Q}{2M} i\sigma_y\right]\right\}$

$$G_M^N(Q^2) = -\frac{P^+}{M} Tr\left\{ J^y \, i\sigma_z \right\}$$

(q along x-axis)



Λ = 0.3 GeV

m_Q = 0.25 GeV

r_Q = 0.21 fm

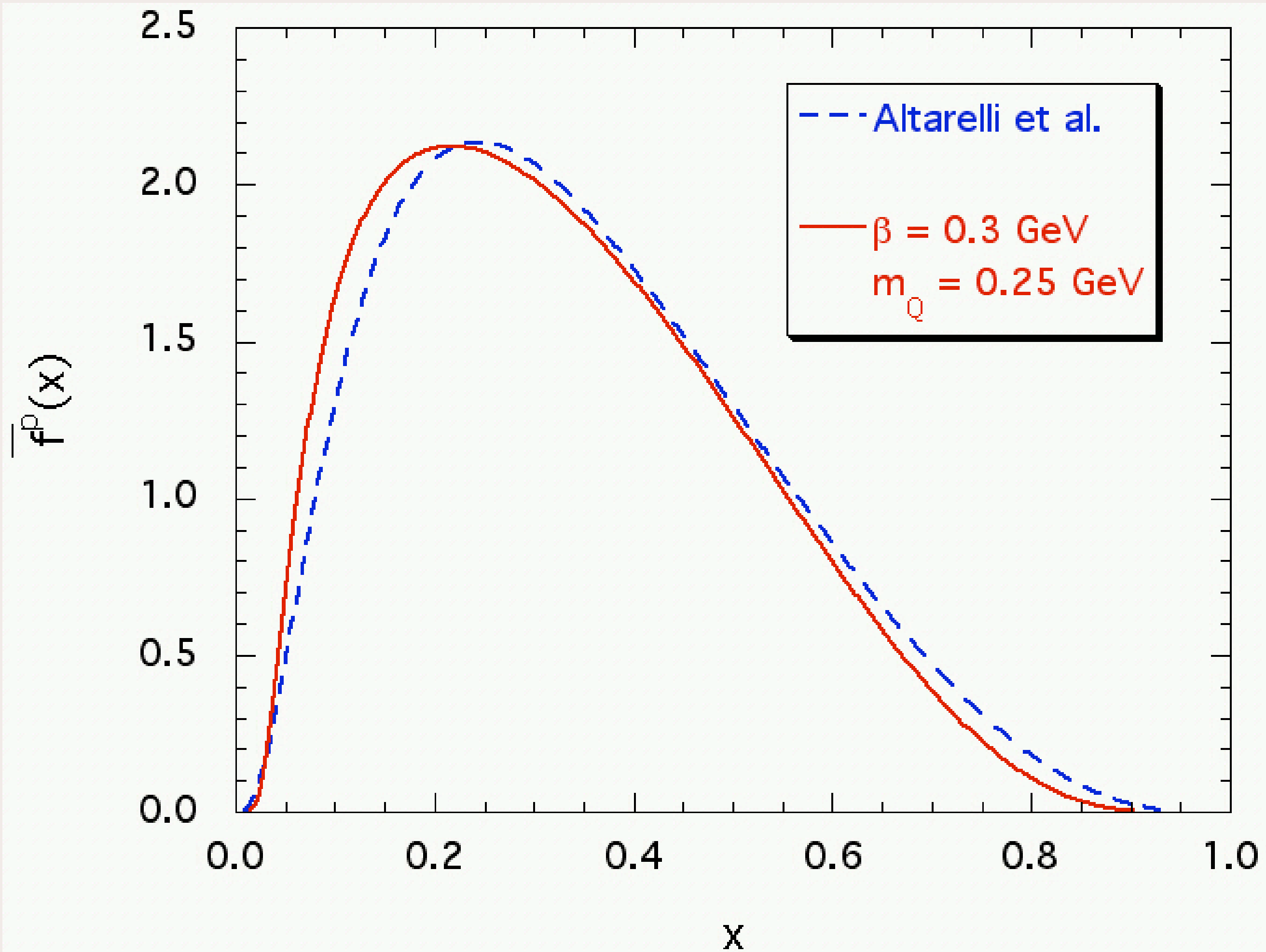
r_Q = 0.33 fm

same Λ and m_Q

just one possibility !

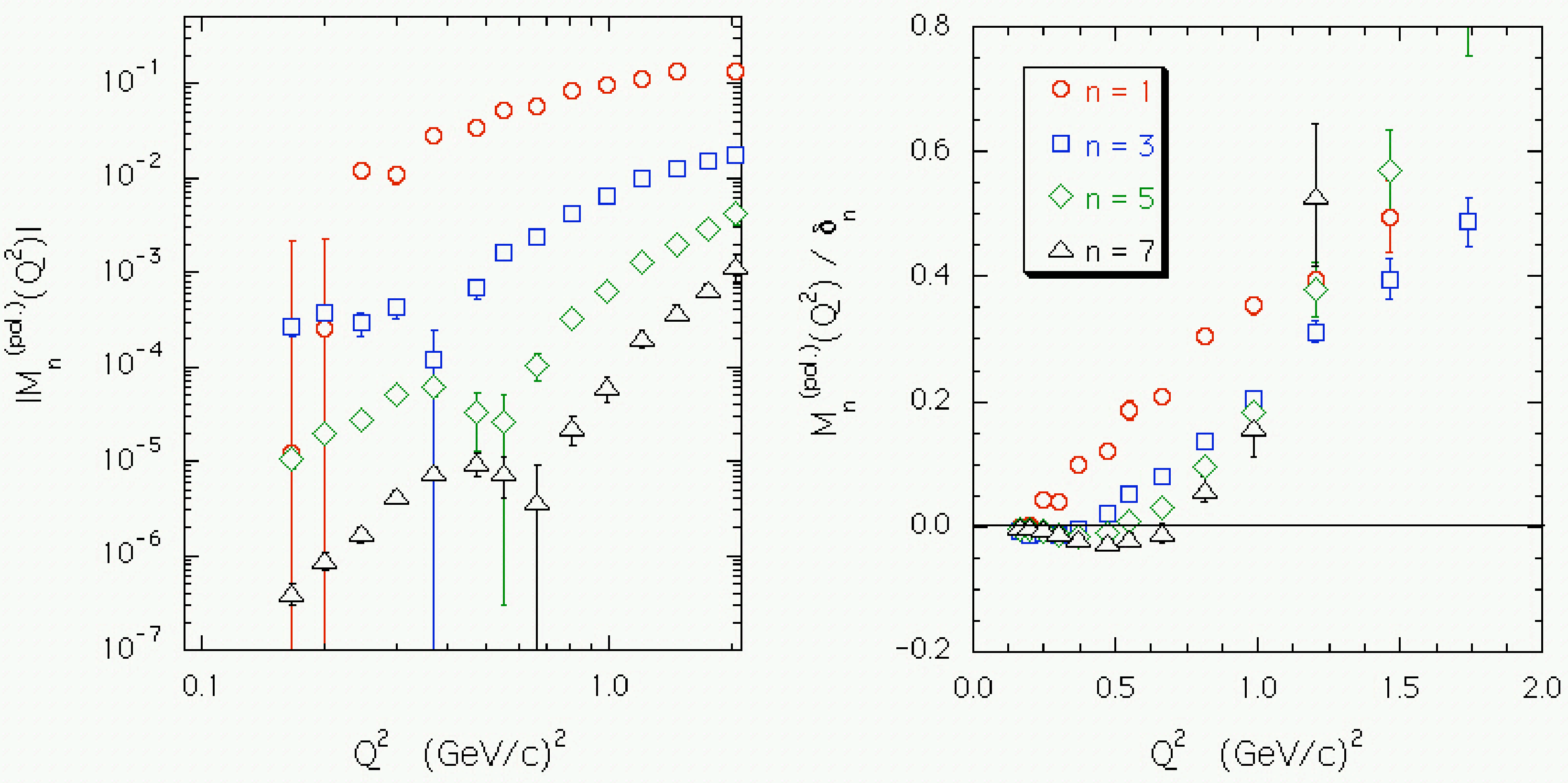
- generalized two-stage model: $F_2^p\left(x,Q^2\right) = \int_x^1 dz\, \bar{f}^p(z)\, F_2^{CQ}\left(\frac{x}{z},Q^2\right) + \left[F\left(Q^2\right)\right]^2 x \cdot \bar{f}_j^p(x)$

it should be the same distribution !



consistency !

Preliminary Results on Polarized Nachtmann Moments (> 70% from CLAS data)



~ SCALING for n > 1 !!!

$$\delta_n = \frac{5}{18}\left(\frac{1}{3}\right)^{n-1}$$

SUMMARY

- extension of the two-stage model to low values of Q^2 below and around the scale of □SB

  - inclusion of the **elastic coupling** at the CQ level → new **scaling** property

- results of the analysis of the new **CLAS** data for Q^2 between ~ 0.1 and ~ 2 GeV^2:

  - the scaling property is well satisfied by **CLAS** data
  - the CQ form factor extracted from inelastic proton data is consistent with the one required explain elastic nucleon data
  - the constituent quark size turns out to be ~ 0.2 ÷ 0.3 fm.

the inclusive proton structure function at low momentum transfer originates mainly from the elastic coupling with extended objects inside the proton

CONCLUSIONS

CQ's as quasi-particles: dressing of valence quarks with gluons and $q\bar{q}$ pairs

CQ's as intermediate structures between current quarks and hadrons

two-stage model: hadrons are composed by a finite number of CQ's having a structure

consistency with DIS data and first evidence from **CLAS** data at low momentum transfer

the light-front formalisms at $q^+ = 0$ is presently the most suitable approach for developing a **relativistic CQ model**

open problems: 1) baryon spin-orbit puzzle;
2) d/u puzzle at large x.

running and planned experiments at JLab (including its upgrade to 12 GeV) are expected to shed further light on hadron and CQ structures